May 2017 subject reports

Further Mathematics HL

Overall grade boundaries

Higher level

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 14	15 - 29	30 - 42	43 - 52	53 - 62	63 - 72	73 - 100

Higher level paper one

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 22	23 - 45	46 - 66	67 - 81	82 - 96	97 - 111	112 - 150

The areas of the programme and examination which appeared difficult for the candidates

Many candidates were unable to deal with a question involving the theory of convergence of series.

Many candidates are not comfortable dealing with questions on coordinate geometry.

The presentation of proofs involving groups is often poor.

Many candidates are not confident when dealing with vectors in both geometry and linear algebra.

The interpretation of confidence intervals seems to be difficult for most candidates.

The areas of the programme and examination in which candidates appeared well prepared

Most candidates are able to use their calculator to carry out hypothesis tests although not all candidates seem to realise that the output contains more than just the p-value.

Most candidates are confident in dealing with problems involving linear combinations of normal random variables.

Maclaurin series are well understood by the majority of candidates.

Many candidates are confident dealing with graph theory.

The strengths and weaknesses of the candidates in the treatment of individual questions

Q1 – This question was well answered by most candidates. It was however strange to see that many candidates calculated the mean and variance estimates using the formulae and then went on to use the calculator software to carry out the *t*-test, apparently not aware of the fact that the output from the *t*-test included these estimates. A fairly common error was to divide by n instead of $n - 1$ when estimating the variance.

Q2 – Solutions to (a) were often disappointing with some candidates failing to reproduce what was thought to be standard bookwork. The following solution was not uncommon.

$$\begin{aligned} ax &= b(\text{mod } p) \\ x &= \frac{b}{a}(\text{mod } p) \\ &= \frac{b}{a} \times a^{p-1}(\text{mod } p) \text{ using Fermat's Little Theorem} \\ &= a^{p-2}(\text{mod } p) \end{aligned}$$

This is of course incorrect because $\frac{b}{a}$ in line 2 is not an integer. Part (b) was well answered.

Q3 – This was well answered in general although some candidates took a page or more before coming up with the correct solution to (a). Candidates should be aware that systematic reduction of the matrix to triangular form will always lead to a quick solution. Parts (b) and (c) were well answered in general.

Q4 – This question was well answered in general with few candidates miscalculating the variances in the two cases.

Q5 –Solutions to (a) were seen using the Comparison Test or the Limit Comparison Test although solutions were often incomplete with some of the requirements for convergence omitted so that only a minority scored full marks in (a). Some candidates attempted, incorrectly, to use the Ratio Test.

Q6 – Parts (a) and (b) were well answered in general although some candidates failed to justify the order of *P*. In (c), some candidates appeared not to understand that *P* could generate a group.

Q7 – Part (a) was very well answered with some candidates showing off their knowledge by writing the function in the form $\frac{1}{2}(\cosh x+\cos x)$. The application to the Poisson distribution in (b) was not so well answered.

Q8 – Part (a) caused problems for some candidates who obtained the derivative in the form $\frac{2a}{y}$ and then failed to substitute for *t*. Part (b) was reasonably well answered although those candidates who chose to use Pythagoras' Theorem often made algebraic errors.

Q9 – Part (a) was very well answered. Solutions to part (b)(i) were often disappointing with some candidates making sign errors in the application of integration by parts. In (b)(ii), some candidates used repeated integration by parts, not realising that it would be quicker to use the recurrence relation found earlier.

Q10 – Solutions to (a) were often disappointing with many candidates failing to take note of the requirement that the determinant is equal to 1 so that many candidates simply showed that non-singular matrices form a group under multiplication which was not what was asked. Solutions to (b) were generally better although many candidates failed to show that the inverse of a matrix in *H* also belonged to *H*.

Q11 – Parts (a) and (b) were reasonably well answered but (c) caused problems for some candidates. In (c)(i), the explanations were sometimes incomplete and in (c)(ii) some candidates showed that *K ,* and not its complement, was not bipartite.

Q12 – Solutions to (a) were somewhat variable with some candidates giving a correct solution in just a few lines and others taking a page or more to prove the result. Explanations as to why the result showed that the medians are concurrent were often unconvincing with the word symmetrical rarely seen. Surprisingly, many candidates failed to attempt (b) although those who did were usually successful.

Q13 – It has been seen in the past that most candidates do not understand what is meant by a confidence interval and this question simply reaffirmed that view. It would appear that we have to accept the fact that candidates are extremely efficient in finding confidence intervals using a calculator but do not really understand what it is that they have found. The most common response to (b)(ii) was along the lines of 'the confidence interval which the teacher has calculated contains μ with probability 0.95' which is simply repeating the statement which the question described as incorrect.

Q14 – Part (a) was reasonably well answered. Solutions to (b), however, were often poorly presented and difficult to follow with many candidates failing to follow the correct logical steps to arrive at the solution.

Q15 – Correct solutions to (a) were rarely seen with most candidates having no idea how to use scalar products to obtain the required result. Candidates were generally more successful with (b).

Recommendations and guidance for the teaching of future candidates

Many candidates seem to be unaware of the instruction on the front of the examination paper which states that 'Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures'. Many candidates lose marks by failing to obey this instruction.

Although candidates are generally competent in using their graphical calculators, not all candidates use them efficiently. Candidates should be aware that the output from carrying out a hypothesis test contains not only the value of the test statistic and the p-value but also the means and variances and degrees of freedom.

The presentation of theoretical problems is often poor and candidates should be encouraged to write these solutions in a logical order.

Higher level paper two

Component grade boundaries

Grade:	1	2	3	4	5	6	7
Mark range:	0 - 20	21 - 41	42 - 60	61 - 75	76 - 90	91 - 105	106 - 150

General comments

Further Mathematics is a very challenging course: The content includes that for the four Mathematics options together with Geometry and Linear Algebra; candidates need to bring to mind this considerable amount of mathematics during each of the two written assessment papers. It is astonishing that so many candidates are successful.

The areas of the programme and examination which appeared difficult for the candidates

Given that candidates come with different experiences with regard to their Mathematics HL options choices, questions which are difficult for some, are quite routine to others. Generally, however, Geometry and the more abstract aspects of Linear Algebra are found to be difficult. Even for those candidates who are technically competent over wide areas, weaknesses show up in the proper logical structure of abstract proofs.

The areas of the programme and examination in which candidates appeared well prepared

The application of Dijkstra's algorithm; the terminology and mechanics of equivalence relations; routine recurrence relations; the use of a GDC in matrix computations; the integrating factor method for solving a first order linear differential equation; the group axioms.

The strengths and weaknesses of the candidates in the treatment of individual questions

Q1 (a)(i) Often poorly expressed, few candidates clearly distinguished between the criterion for an Eulerian trail from that for an Eulerian circuit.

Q1 (a)(ii) Usually OK, but occasionally a Hamiltonian circuit was given.

Q1(b) The most common answer involved an annotated graph. Some candidates failed to observe the instruction 'Your solution should show clearly that this algorithm has been used.' It was hard to allocate marks if the graph resulted in an incorrect total weight (often 15).

Q2 (a) Generally adequately answered, although some assumed *x* and *y* belonged to *H*. Some candidates seemed unaware of the distinction between 'equals' and 'implies'; thus $xRy = yRx$ is unacceptable as part of the proof that *R* is symmetric.

Q2(b) (i) Generally well done. (ii) Many candidates incorrectly put 3 and 9 in the same class.

Q3(a)(i)(ii) Generally well done. A small number of candidates used strong induction.

Q3(a)(iii) The first three marks were often awarded, but only the best candidates were able to deal with the limit.

Q3(b) The first two marks were usually awarded. Working back from the characteristic values to the characteristic equation and thence the recurrence equation defeated all but the best candidates.

Q4(a) Generally well done.

Q4(b)(ii) This tended to be all or nothing.

Q4(c) Some didn't work out B^2 explicitly. The main marking problem was that 8 and *B* were very

hard to distinguish between on many scripts.

Q5(a)(i) Nearly always answered successfully, but many made a meal of it using sine and cosine.

Q5(a)(ii) A few went in the wrong direction in their integration by parts.

Q5b The majority of candidates were familiar with the integrating factor method of solving first order linear differential equations.

Q6(a) and (b)(i) Usually well done.

Q6(b)(ii) Here the most common error was to assume that the angle CAD or ABC was a right angle.

Q7 (a)(i)(ii) The most common error was to omit the power of *t*, so the result was not a power series.

Q7(b)(ii) Often well done. The most common error was to think that the derivative of ln4 is $\frac{1}{4}$.

Q8 (a) Quite hard to mark, because most candidates failed to handle the notations of primality and divisibility in a convincing logical way.

Q8(b) Generally $\frac{3}{4}$ because few candidates were able to deal with the existence of inverses.

Q8(c) (ii) Few stated that 2 divides (*n*+1) because *n* is odd.

Q9 (a) Many helpfully used diagrams to support their reasoning.

Q9 (b)(I) Those who read the question, generally gained full marks. Some diagonalized the matrix, with varying degrees of success.

Q9(b)(ii) Candidates who observed that $\cos\alpha = 2\sin\alpha$, nearly always verified the absence of the crossterm.

Q9(iv) Many candidates stated correctly the coordinates of the rotated hyperbola, but few even attempted to find those of the original conic.

Recommendations and guidance for the teaching of future candidates

Impress on students the importance of the use of well supported logical arguments in all aspects of mathematics.